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Fuzzy Dynamic Programming, Fuzzy Adaptive Neuro Control, and the General Medical Diagnosis Problem

A. O. ESOGBUE
School of Industrial and Systems Engineering
Georgia Institute of Technology
Atlanta, GA 30332-0205, U.S.A.

Abstract—The fuzzy medical diagnosis decision models of Esogbue and Elder employed fuzzy sets theory to directly and more correctly model all the information nets useful in reaching a scientific understanding of a patient's health status. This knowledge was utilized in constructing a sensible medical hypothesis decision problem. In this paper, we show how fuzzy dynamic programming and neural networks can be used to extend the process to other phases by linking the four stages of medical hypothesis, physician's observation, preliminary diagnosis, and final diagnosis in an adroit manner.

Keywords—Fuzzy dynamic programming, Fuzzy sets, Medical diagnosis, Fuzzy-neuro controllers.

1. INTRODUCTION

The problem posed in [1] is to develop a model useful in treating the medical hypothesis decision problem. As is evident, this problem is in reality, a subproblem of the entire medical diagnosis decision problem which consists of two interdependent phases. This is shown diagrammatically in the flowchart of Figure 1.

Our emphasis then was on the development of mathematical models which employ fuzzy set theory in modeling specific diagnostic decision protocols. These decisions involve the use of diagnostic information and the reaching of effective medical hypotheses, preliminary, and final diagnoses. Each component requires some or all of the following fuzzy information nets:

(i) patient's past history H;
(ii) medically designated symptoms A;
(iii) signs observed by the physician S; and
(iv) results of clinical and diagnostic tests Z.

For example, medical hypothesis requires the two information matrices $\langle H, A\rangle$, initial preliminary $\langle H, A, S\rangle$, other preliminary diagnosis $\langle H, A, S, Z\rangle$, and final diagnosis $\langle U, A, S, Z\rangle$. We focused on the medical hypothesis subsystem and developed a computerized model for its implementation which showed vast improvement over computerized diagnoses relying exclusively on Bayesian models.

To be more effective, it is desirable to integrate the foregoing phases into a single cohesive model. Thus, our current problem is to devise a method to be used in linking up these hitherto

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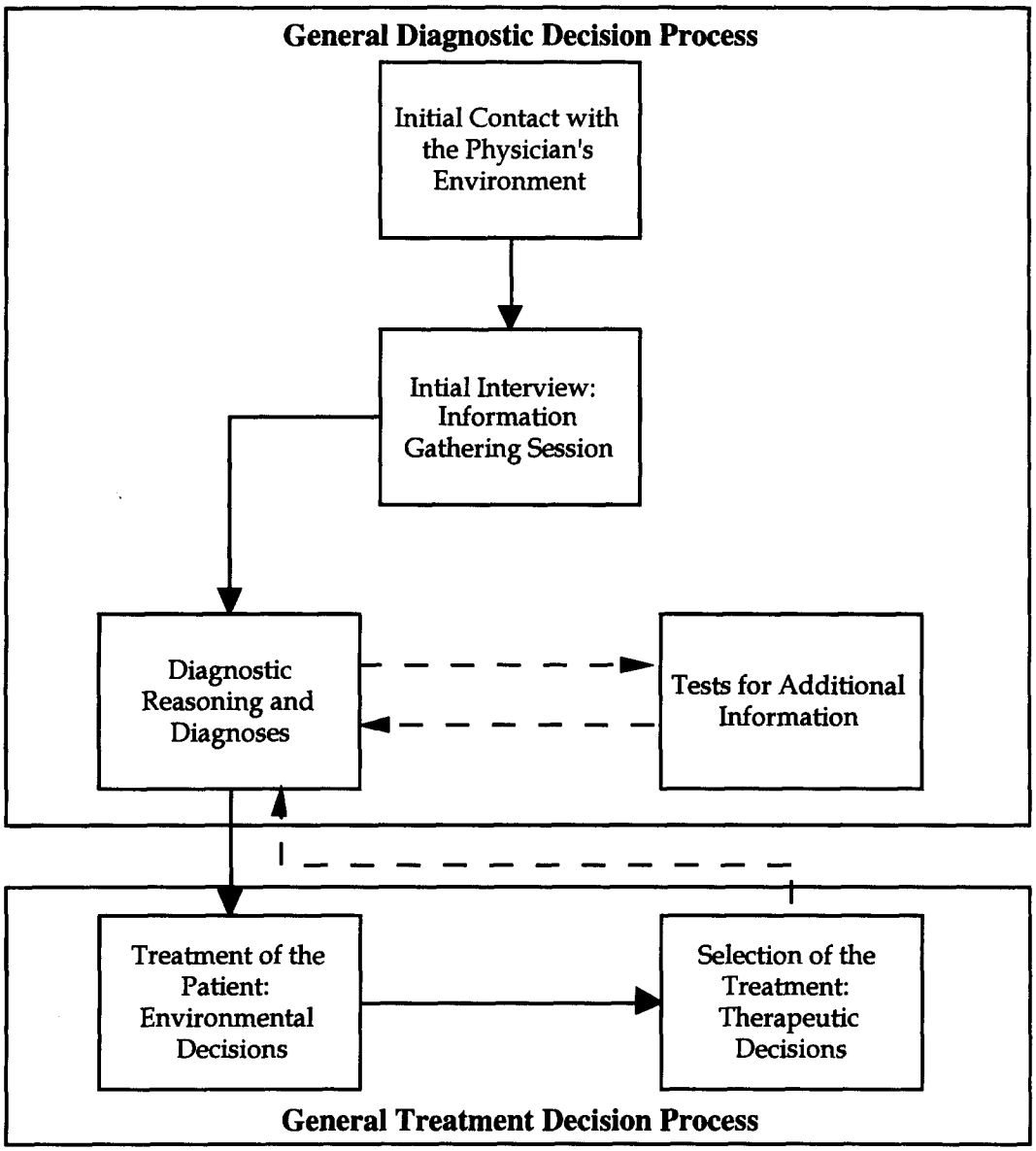

Figure 1. Systems diagram of the general diagnostic and treatment processes.

disparate phases in an adroit manner. Dynamic programming, but in particular, fuzzy dynamic programming [2], offers the most natural and optimal procedure for accomplishing this task. The rest of the paper is organized as follows: in Section 2, the general dynamic programming model for the process is briefly introduced. In Section 3, we particularize it to our problem. Section 4 deals with the fuzzy computational and data generation issues. The paper is concluded in Section 5 with a brief sketch of the attendant computational issues and a discussion of the use of fuzzy-neural networks as well as novel tools of intelligent control [3] in mitigating these problems. This enhances the implementation and on-line usage of the proposed models.

2. THE LINKAGE PROBLEM AS A FUZZY DYNAMIC PROGRAM

Let us now sketch the fuzzy dynamic programming concept which is fundamental to our development. We assume that the system under control A, is an N-stage process in which

state X_i, $i = 1, \ldots, N$, ranges over a finite set $X = \{\sigma_1, \sigma_2, \ldots, \sigma_n\}$, and the decision U_i ranges over a finite set $U = \{\sigma_1, \sigma_2, \ldots, \sigma_m\}$. The state transformation equation is governed by

$$X_{i+1} = f(X_i, U_i), \qquad i = 0, 1, 2, \ldots, N, \tag{1}$$

where $f : X \times U$ to X with a membership function of the form $\mu(X_{i+1})$. Here it is assumed that f is not a fuzzy function, but if it were, the membership functions would be of the form $\mu(X_{i+1} \mid X_i, U_i)$. At each stage i, the decision is subject to a fuzzy constraint C^i which is a fuzzy set in U with membership function $\mu_i(U_i)$. Also, we assume that the goal is a fuzzy set G^N in X characterized by a membership function $\mu_{G^N}(X_N)$. We may define the functional as

$$\begin{aligned}\mu_{G^N}(X_N) = {} & \text{maximum membership functional value for a specified} \\ & \text{goal } G^N \text{ attained by operating optimally over stages} \\ & N-1, \ldots 1, 0 \text{ and given the final state } X_N,\end{aligned}$$

and can readily derive the following recursive relation in the spirit of Bellman and Zadeh [4]:

$$\mu_{G^{N-u}}(X_{N-u}) = \max_{U_{N-u}} \left\{ \min_u \left[\mu_{N-u}(U_{N-u}), \mu_{G^{N-u+1}}(X_{N-u+1}) \right] \right\}, \tag{2}$$

$$u = 1, \ldots, N. \tag{3}$$

The boundary condition is expressed in terms of the given values for $\mu_{G^N}(X_N)$. Also

$$U_i = \Pi_i(X_i), \qquad i = 0, 1, \ldots, N-1, \tag{4}$$

where Π is the policy function, and the transition equation

$$X_{i+1} = f(X_i, U_i), \qquad i = 0, 1, \ldots, N-1. \tag{5}$$

These recurrence equations will be employed to generate the set of maximizing decisions $U_0^*, U_1^*, \ldots, U_{N-1}^*$. In the general case assumed here [4], we have a deterministic system under control whose dynamics is described by a state transition equation

$$X_{i+1} = f(X_i, U_i), \qquad i = 0, 1, \ldots, \tag{6}$$

where $X_i, X_{i+1} \in X = \{s_1, \ldots, s_n\}$ are the states (equated here, for simplicity but without loss of generality, with outputs) at time (control stage) i and $i+1$, respectively, and $U_i \in U = \{c_1, \ldots, c_m\}$ is the control (input) at i; X and U are assumed finite throughout this paper.

At each i, U_i is subjected to a fuzzy constraint $\mu_{C^i}(U_i)$ with a fuzzy goal $\mu_{G^{i+1}}(X_{i+1})$ imposed on X_{i+1}.

The performance of the multistage decision making (control) process is evaluated by the fuzzy decision which, assumed to be a decomposable fuzzy set in $U \times X \times \cdots \times U \times X$, is

$$\begin{aligned}\mu_D(U_0, \ldots, U_{N-1} \mid X_0) &= \mu_{C^0}(U_0) \wedge \mu_{G^1}(X_1) \wedge \mu_{C^{N-1}}(U_{N-1}) \wedge \mu_{G^N}(X_N), \\ &= \bigwedge_{i=0}^{N-1} (\mu_{C^i}(U_i) \wedge \mu_{G^{i+1}}(X_{i+1})),\end{aligned} \tag{7}$$

where $X_0 \in X$ is an initial state, X_{i+1}'s are given by (5), and N is the termination time (fixed and specified in the basic case).

We seek an optimal sequence of controls $U_0^*, \ldots, U_{N-1}^*$ such that

$$\begin{aligned}\mu_D(U_0^*, \ldots, U_{N-1}^* \mid X_0) &= \max_{U_0,\ldots,U_{N-1}} \mu_D(U_0, \ldots, U_{N-1} \mid X_0) \\ &= \max_{U_0,\ldots,U_{N-1}} \bigwedge_{i=0}^{N-1} (\mu_{C^i}(U_i) \wedge \mu_{G^{i+1}}(X_{i+1})).\end{aligned} \tag{8}$$

For simplicity, it is often assumed (in principle also here) that at each i, fuzzy constraints are given, and a fuzzy goal is only imposed on the final state. Then, the fuzzy decision is

$$\mu_D(U_0, \ldots, U_{N-1} \mid X_0) = \mu_{C^0}(U_0) \wedge \cdots \wedge \mu_{C^{N-1}}(U_{N-1}) \wedge \mu_{G^N}(X_N) \tag{9}$$

and $U_0^*, \ldots, U_{N-1}^*$ is sought such that

$$\mu_D(U_0^*, \ldots, U_{N-1}^* \mid X_0) = \max_{U_0,\ldots,U_{N-1}} (\mu_{C^0}(U_0) \wedge \cdots \wedge \mu_{C^{N-1}}(U_{N-1}) \wedge \mu_{G^N}(X_N)). \tag{10}$$

The foregoing is quite general and perhaps the best known version of this model. As is evident, it assumes a deterministic system under control whose dynamics is described by the state transition equation given in equation (1). In virtually all cases (of a particular termination time and system under control) a dynamic programming type algorithm can be devised; the first computational algorithm for the implementation of fuzzy dynamic programming models can be found in [5]. For a comprehensive and didactic treatment of fuzzy dynamic programming, see [2,6].

3. APPLICATION TO THE MODEL OF ESOGBUE AND ELDER [1]

We now wish to illustrate an application of the above formulation to the diagnosis decision model proposed by Esogbue and Elder [1]. The problem is to diagnose the disease(s) of a patient assuming that we have the following fuzzy information matrices H, A, S, and Z given. First, we look at the resultant mathematical program as if it were not fuzzy, and then make adjustments. As may be evident, we consider the problem as decomposable into the following four stages of dynamic programming.

Stage 1. Medical hypothesis from patient's history.
Stage 2. Physician's observation.
Stage 3. Preliminary diagnosis.
Stage 4. Final diagnosis.

To illustrate, we will start at Stage 1 and work forward.

Let $(\sigma_1^1, \sigma_2^1, \ldots, \sigma_{n_1}^1)$ be a subset of X. The *patient's past history* is represented by $H = [h(1), h(2), \ldots, h(m)]$, with m the total number of relevant aspects for the M diseases under consideration. Here σ_j^1 is an m-vector corresponding to the fuzzy matrix H. The grade of membership for state σ_j^1 in disease cluster k is $\mu_k(\sigma_j^1)$. The value of $\mu_k(\sigma_j^1)$ can be obtained in many ways. A formal approach is via an optimal fuzzy clustering algorithm of the type proposed by Esogbue [7] or the more user friendly variation advanced recently by Liu and Esogbue [8]. Alternatively, it can be approximated by the physician, derived from a mathematical formula, or generated via fuzzy neural networks. These will be discussed in detail later. One reasonable formula seems to be

$$\mu_k(\sigma_j^1) \left(|\sigma_j^1|^{-1} [\beta_1, \beta_2, \ldots, \beta_m]^\top \right) m^{-1}, \tag{11}$$

$$\text{where } 0 \leq \beta_j \leq 1, \quad \forall j \text{ with } k = 1, 2, \ldots, M \tag{12}$$

and M, the number of diseases. Note that β_j represents the importance of factor j in diagnosing disease k. ($\beta_j = 0$ iff symptom j has nothing to do with disease k.)

To select a particular set of diseases or eliminate diseases from consideration, we need a policy. The determination of this policy is a consequence of a dynamic programming solution.

Let $\Pi(\sigma_j^1) = \{\sigma_k : \mu_k(\sigma_j^1) \geq \phi_1\}$, where $\{\sigma_k\}$ are the diseases to suspect in the patient and ϕ_1 is some specified threshold. These thresholds are usually medically known or designated for certain diseases and patient profiles. Although they may in general be fuzzy, we assume as in practice, that they are precisely determined from previous medical knowledge or experience. We note that the policy could yield multiple diseases to suspect and should also narrow the possibilities.

The next step is for the *physician to observe* signs $S = [S(1), S(2), \ldots, S(f)]$ where f is the number of possible signs for the diseases of concern. Let $\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_{n_2}^2\} \in X$. Now $S(\cdot) \in [0,1]$ is specified. So the interval $[0,1]$ will be split into subintervals. Say for instance: $[0, 1/4), [1/4, 3/4), [3/4, 1)$, and σ_j^2's will reflect the possible combinations of each $S(\cdot)$ at different levels. The membership function $\mu_k(\sigma_j^2)$ then needs to be calculated and the policy $\Pi(\sigma_j^2) = \{\sigma_k : \mu_k(\sigma_j^2) \geq \phi_2)$ can be attained. Here, again multiple diseases $\{\sigma_k\}$ may be suspected and the set of possibilities may be lessened. As usual, $\mu_2(\sigma_j)$ is needed when fuzziness is considered.

Next, the *preliminary diagnosis* needs to be made. This is the third stage of the dynamic programming process. Clinical and diagnostic tests to check for the diseases $\{\sigma_k\}$ are run and $Z = [Z(1), Z(2), \ldots, Z(k)]$ with Z equal to the number of tests performed on patient i and $Z(i) \in [0,1]$.

Again, since $Z(i) \in [0,1]$, some intervals need to be chosen for each $Z(i)$, $i = 1, \ldots, k$. Then the state σ_j^3 can be matched with Z to get $\mu_k(\sigma_j^3)$. Also, a policy $\Pi(\sigma_j^3)$ should be established. Let $\Pi(\sigma_j^3) = \{\sigma_k : \mu_k(\sigma_j^3) \geq \phi_3\}$.

Stage 4 is the *final diagnosis* stage. Here, additional tests may need to be run and/or tests may need to be rerun (the size of Z may increase), so a diagnosis decision $\{\sigma_k\}$ can be resubstituted. The final policy is determined after $\mu_k(\sigma_j^4)$ is evaluated. This leads to a diagnosis decision policy: $\Pi(\sigma_j^4) = \{U : \mu_k(\sigma_j^4) \geq \phi_4\}$ for some specified threshold ϕ_4.

4. FUZZY COMPUTATIONS AND DATA GENERATION

If the problem were not fuzzy, we would have a solution at hand. But since it is, we can now work backwards using the recursive relationship of the dynamic programming functional equation to obtain the values $\mu_k(\sigma_j^i)$ that are necessary. To begin with, we need the following three pieces of data:

(i) ending values $\mu_k(X_4)$ for all possible values of X_4, represented by $\{\sigma_1^4, \sigma_2^4, \ldots, \sigma_k^4\}$;

(ii) $\mu_k(\sigma_k)$ for each stage i and disease k. Recall that this is the membership function value of being able to diagnose a disease k at stage i; and

(iii) a stage transition table showing the relationship $X_{i+1} = f(X_i, U_i)$; this is a very key concept.

Here f was assumed not fuzzy but for particular problems, especially those of a novel type, it may need to be. This assumption states that given state X_i and the decision U_i, the next state will be precisely X_{i+1}. The reasonableness of this assumption, especially in situations where only a small number of entries in $(\sigma_1, \sigma_2, \ldots, \sigma_n)$ are considered reasonable, will need to be estimated, perhaps with a group of physician experts via some Delphi type of exercise. If $f(\cdot, \cdot)$ is fuzzy, implying fuzzy transition mappings, then a technique such as the one proposed by Baldwin and Pilsworth [9] may be invoked. However, computational difficulties experienced with the numerical implementation of this model may necessitate the use of an alternate algorithm such as the one suggested in [6].

We note that much of the required databases consist of information expressed in terms of membership functions $\{\mu(\cdot)\}$. Let us briefly address them as well as instructive methods for their acquisition. In addition to the suggested mathematical formula, they can be derived using expert physicians as in [1] or [10], statistical modeling [11], or via adaptive neural networks such as those we have recently developed [12]. Experimentation with some of these approaches is in progress.

Let us now focus on constructive methods for their generation. In addition to the mathematical formula or equations (11) and (12), other formulas may be utilized. For example, the membership functions can take the particular form of the S-shaped or sigmoid function given by the following equation:

$$\mu_k(\sigma_j^i) \mid (\alpha,\beta,\gamma) = \begin{cases} 0, & \text{for } \sigma_j^i \le \alpha, \\ 2\left(\dfrac{\sigma_j^i - \alpha}{\gamma - \alpha}\right)^2, & \text{for } \alpha < \sigma_j^i \le \beta, \\ 1 - 2\left(\dfrac{\sigma_j^i - \gamma}{\gamma - \alpha}\right)^2, & \text{for } \beta < \sigma_j^i \le \gamma, \\ 1, & \text{for } \sigma_j^i > \gamma. \end{cases} \tag{13}$$

Note that here we have assumed the value of the function $\mu(\cdot)$ for $\sigma_j^i = \beta$ to be equal to 0.5, the midpoint of the interval whereas $\beta = (\alpha + \gamma)/2$, and α, β, and γ are the term sets for a given disease state.

The physician's knowledge or expertise could be used to modify the shape of this function and shift their midpoints to conform with a particular form of symptom or disease progression. Practical examples are the uniform and beta distributions. Modification of any of these can take the form of operators such as concentration, dilation, and contrast intensification corresponding, for example, to the physician's belief or knowledge about the rate of growth or deterioration of a patient's condition over time. These operations are represented mathematically in equation (14):

$$\begin{aligned} &\text{Concentration:} && \mu_{\mathrm{con}(k)}(\sigma) = (\mu_k(\sigma))^2, \\ &\text{Dilation:} && \mu_{\mathrm{dil}(k)}(\sigma) = (\mu_k(\sigma))^{1/2}, \\ &\text{Contrast intensification:} && \mu_{\mathrm{int}(k)}(\sigma) = \begin{cases} 2(\mu_k(\sigma))^2, & \text{for } \mu_k(\sigma) \in [0,.5], \\ 1 - 2(1-\mu_k(\sigma))^2, & \text{otherwise}, \end{cases} \end{aligned} \tag{14}$$

and diagrammatically in Figures 2–5, respectively.

5. NEURAL NETWORKS AS A TOOL FOR MEMBERSHIP FUNCTION GENERATION

The problem with each of the above mathematically or experimentally driven approaches is the presence of considerable subjectivity and, in some cases, their *ad hoc* nature or inappropriateness. There now exist some competitive methods for their amelioration. Let us cite one of them. The use of neural networks can remedy these shortcomings. Here, we may view the degree of activation of a neuron as the degree of membership of a symptom, sign, or test in a particular disease cluster or set (fuzzy). We can construct a neural network which can then be trained to synthesize the physician's descriptors or matchings of various relationships, usually described linguistically, between the various diagnostic parameters and stages. The neural network basically then learns the "if-then" associations. This could be either on-line or off-line, and the training data and/or diagnostic decision rules may be provided as data in the form of input-output pairs by the physician, for example. A limitation of this approach, of course, is the availability of adequate training data coupled with the slow convergence rate of the classical back propagation dependent neural networks.

There now exist an array of these networks as reviewed in [12]. Their utility to the diagnostic process modeling varies. The ones of most interest are those of the self-organizing, adaptive variety. We note that even in the so-called unsupervised networks, some form of supervision usually takes place. In certain diagnosis decision problems, especially those of the novel types, a high degree of uncertainty may exist. This may necessitate the use of neural networks of the type proposed by Esogbue and Murrell [12] and subsequently enhanced by Esogbue and Hearnes [13].

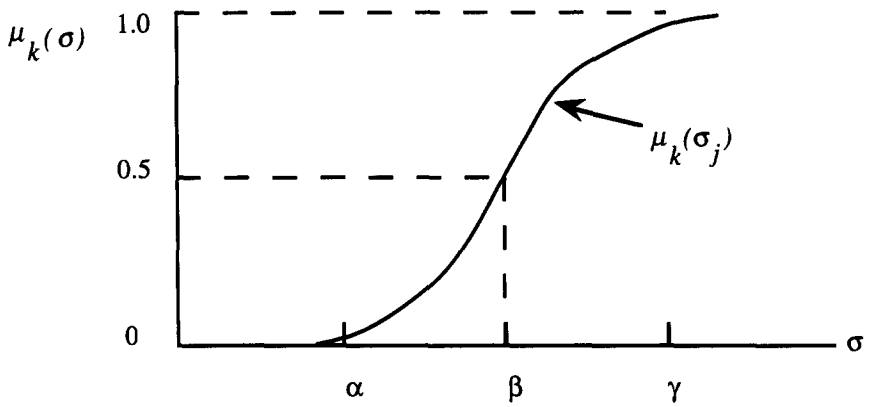

Figure 2. An example of the sigmoid membership function.

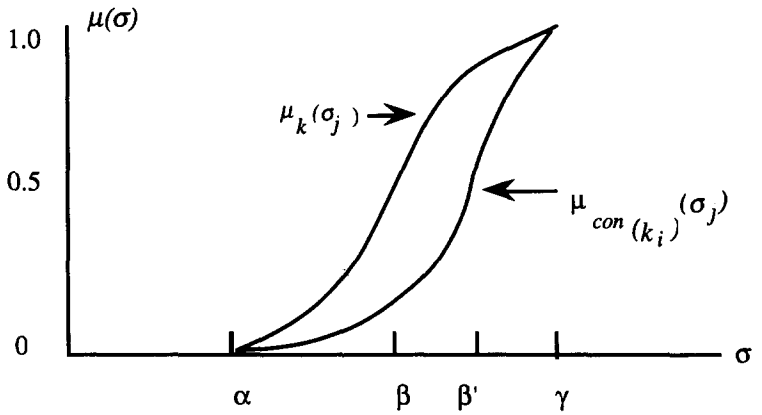

Figure 3. An example of concentration operation.

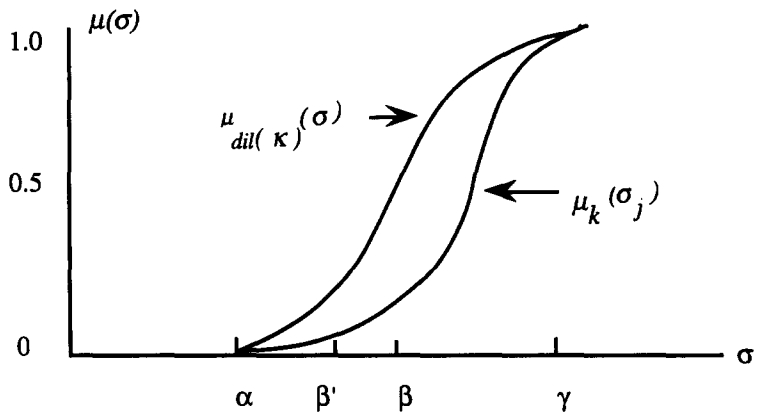

Figure 4. An example of dilation operation.

This is a fuzzy adaptive on-line controller which does not presuppose the existence of a model of the plant or the shape of membership functions. Further, its operation is not contingent on the availability of a set of decision rules or training data.

A diagrammatic representation of the network is given in Figure 6. As can be seen, in addition to the plant which in this case represents the diagnosis decision model, it consists of the following five subsystems or networks.

(i) The Statistical Fuzzy Discretization Network (SFDN) which uses a variation of the Kohonen's self-organizing feature map to fuzzify and aggregate similar plant states thus permitting implementation of the control decision as a discrete relation.

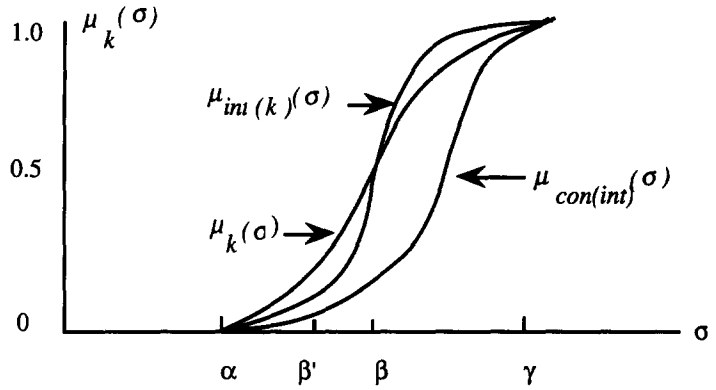

Figure 5. Example of contrast intensification operation.

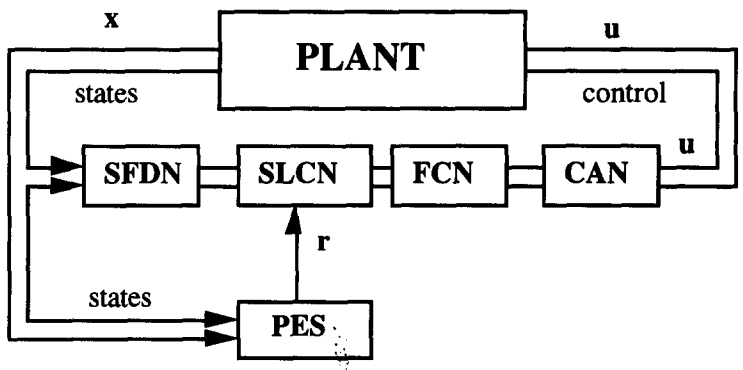

Figure 6. Fuzzy neural network controller subsystems and plant.

(ii) The Fuzzy Correlation Network (FCN) which uses fuzzy associative or correlation network to implement fuzzy control rules as a fuzzy relation.

(iii) The Stochastic Learning Correlation Network (SLCN) which consists of a matrix of nodes with each row corresponding to a particular fuzzy input state and each column to a particular fuzzy control action. This subsystem is used to test and learn the efficacy of pairing a given control vector set with a vector input state using the performance evaluation provided by the performance evaluation subsystem.

(iv) The Control Activation Network (CAN) which defuzzifies a fuzzy control vector input to produce crisp controls or decisions.

(v) The Performance Evaluation Subsystem (PES), akin to the stochastic learning automata of Narendra (see [12]), provides reinforcement signals to the SCLN as a feedback on the effectiveness of the control action or performance.

Note that this controller successfully obviates the problems attendant on backpropagation networks and is very well suited to on-line control decisions. In the current form of the controller, the learning tools feature two variations of dynamic programming learning algorithms in the stochastic mode. Specifically, both the Temporal Difference (TD) and Q-Learning methods have been modified and used in this phase of the controller.

The implementation of the foregoing controller on the physician diagnosis decision problem is the subject of an ongoing inquiry to be reported on later.

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